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METHOD OF A SYNCHRONOUSLY MOVING FIELD AND ITS APPLICATION TO CALCULATING HEAT AND MASS TRANSFER PROCESSES IN A FILTER BED

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An efficient method for calculating the nonstationary fields of moisture contents in drying or adsorption, based on the property of their synchronous movement along a bed of a solid phase, is substantiated.

The processes occurring in a filter bed are characterized by presence of the zones with distinctly differing kinetics and moving boundaries between them. Along with nonlinearity of the initial equations, this circumstance greatly complicates calculations and frequently necessitates a substantial simplification of the picture of the process or the use of insufficiently justified experimental approximations: kinetic approximations, temperature-moisture content ones, etc. [1-8]. Therefore, a search for calculating procedures that are common for all zones and ensure an improved accuracy is rather urgent. The method put forth below is one of such solutions.

The absence of contact mass transfer over the height of a bed of a solid phase with moisture transfer effected only by a gas flow is taken as the basic assumption. Then, having isolated the element of thickness dh in the bed, we will have the equation of material balance (a periodic process)

$$w\rho_g \frac{d_h x}{dh} = -\rho_s (1-\varepsilon) \frac{d_\tau u}{d\tau} - \rho_g \varepsilon \frac{d_\tau x}{d\tau}. \tag{1}$$

Here, the subscripts h and τ on the differentials signify a partial increment of the parameter along the corresponding coordinate. Having discarded, because of smallness, the second term in the right side of Eq. (1), we write it in the following form:

$$\frac{dh}{d\tau} = -\frac{w\rho_g}{\rho_s (1-\varepsilon)} \frac{d_h x}{d_\tau u}. \tag{2}$$

The concentration (moisture content) field over the bed height along the coordinate h. We distinguish, in the bed, two parallel planes (fronts) a distance dh from each other. Let the moisture content of the lower front be u and that of the upper front, $u + d_h u$. The time interval $d\tau$ is chosen such that the moisture content of the upper front is reduced by $d_h u$. The latter is equivalent to the movement of the lower front over the distance dh. Then, the ratio $dh/d\tau$ in Eq. (2) represents the front velocity. On the other hand, the moisture content of the front decreased over the time $d_\tau u$, and therefore, it can be written that

$$d_h u = -d_\tau u. \tag{3}$$

In view of Eq. (3) we obtain an equation for the velocity of the movement of the fronts of moisture content along the coordinate h:

$$v = \frac{w\rho_g}{\rho_s (1-\varepsilon)} \frac{d_h x}{d_h u} = \frac{w\rho_g}{\rho_s (1-\varepsilon)} \left(\frac{\partial x}{\partial u}\right)_\tau. \tag{4}$$

The derivative in the right side of Eq. (4) is expressed as

$$\left(\frac{\partial x}{\partial u}\right)_\tau = \frac{\partial x}{\partial h} \Big/ \frac{\partial u}{\partial h}. \tag{5}$$

Then Eq. (4) takes the form

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$$v \frac{\partial u}{\partial h} = \frac{w\rho_g}{\rho_s (1 - \varepsilon)} \frac{\partial x}{\partial h}. \tag{6}$$

We now integrate with respect to h assuming in the general case $v = v(h)$:

$$\int v \frac{\partial u}{\partial h} dh = \frac{w\rho_g}{\rho_s (1 - \varepsilon)} x + C(\tau). \tag{7}$$

The integration in Eq. (7) can be done by parts:

$$vu - \int u \frac{\partial v}{\partial h} dh = \frac{w\rho_g}{\rho_s (1 - \varepsilon)} x + C(\tau). \tag{8}$$

To determine $C(\tau)$, we use the following condition at the lower boundary (LB) of the bed at the gas entry: at $h = 0$, $x = x_0$, $u = \dot{u}$, and $v = \dot{v}$. Then Eq. (8) takes the form

$$vu - \frac{w\rho_g}{\rho_s (1 - \varepsilon)} (x - x_0) = \dot{v}\,\dot{u}. \tag{9}$$

Because u and x generally vary over the bed height monotonically (both of them increase), v should also be a monotonic function of h, without extrema. The following conditions occur at the upper, fairly remote boundary of the bed: $u = u_0$, $x = x_{sat}$, and $v = v_\infty$. Then we obtain a relation between the velocities of the fronts at the boundaries:

$$v_\infty = \frac{1}{u_0} \left[\dot{v}\,\dot{u} + \frac{w\rho_g}{\rho_s (1 - \varepsilon)} (x_{sat} - x_0) \right]. \tag{10}$$

Consideration is next given to the case of equal velocities of the fronts, when $\dot{v} = v_\infty = \bar{v}$, which is of great practical importance. From Eq. (10) we have

$$\bar{v} = \frac{w\rho_g}{\rho_s (1 - \varepsilon)} \frac{x_{sat} - x_0}{u_0 - \dot{u}}. \tag{11}$$

We now analyze the feasibility of such a pattern of the field movement. Let $\bar{v} = \dot{v}m$ and $\dot{v} = \bar{v}/m$, and then from Eqs. (10) and (11) it follows that

$$v_\infty = \bar{v} \left[1 - \frac{\dot{u}}{u_0} \left(1 - \frac{1}{m} \right) \right]. \tag{12}$$

At the initial instants of drying $\dot{u} \approx u_0$, and then it follows from Eq. (12) that $v_\infty = \dot{v}$, i.e., the fronts move in synchronism.

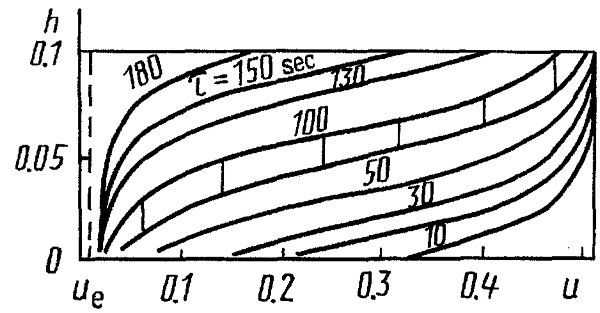
A similar pattern is observed when $\dot{u} \to u_e$ at the lower boundary of the bed. The ratio $\dot{u}/u_0$ usually differs little from zero because of the smallness of u_e, and then from Eq. (12) it also follows that $\dot{v} \approx v_\infty$. Since $\dot{u}$ hardly changes subsequently, the drying zone with all points moving at the same time-independent velocity moves up the bed. This period, taking up a significant part of the total drying time, can rightfully be referred to as steady.

A nonsynchronous movement is also possible in principle in the interval between the processes considered; however, this needs strong causes. Let $\dot{u}/u_0 = 0.5$, and then from Eq. (12) we obtain

$$v_\infty = 0.5\,\dot{v}\,(m + 1). \tag{13}$$

The running velocity of a front is specified not only by the drying mode characterized by the quantity $d_h x$ in Eq. (4) but also by the running profile of the moisture content in the bed (by $d_h u$). Both of them generally correlate so that Eq. (11) is fulfilled but the possibility of its violation must not be ruled out.

For example, with a drastic increase in the gas velocity or temperature, in accordance with Eq. (11), the average velocity of the fronts $\bar{v}$ grows, $d_h x$ at the entry increases, but $d_h u$ does not manage to rise in proportion to $d_h x$ because the moisture content field in the solid phase has inertia, which results in $d_h x / d_h u > (x_{sat} - x_0)/(u_0 - \dot{u})$, i.e., $\dot{v} > v$ and $m < 1$, at the entrance to the bed. Then from Eq. (13) it follows that $v_\infty < \dot{v}$, i.e., the upper fronts move more slowly than the lower ones, and the drying zone shrinks. During shrinking, the slope of the moisture content profile increases, the value of $d_h u$ rises, and the disturbed correlation is gradually restored.



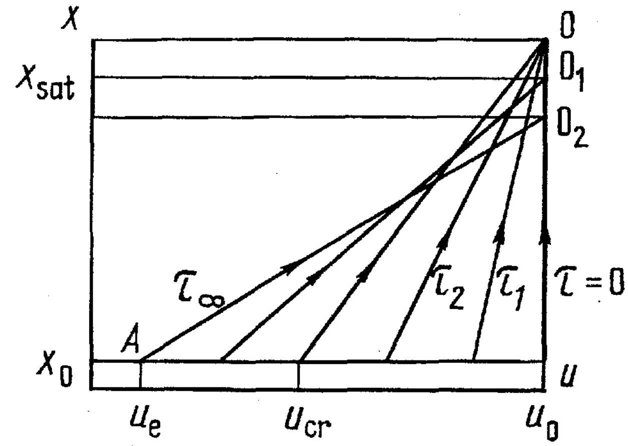


Fig. 1. Moisture content distribution over the bed height (m) for a period of a falling rate.

Fig. 2. Operating lines of convective drying in the coordinates $x - u$.

Likewise, with a decrease in the drying rate at the entrance $\dot{v} < \bar{v}$, $m > 1$, and $v_\infty > \dot{v}$, i.e., the drying zone will expand for some time until $d_h u$ decreases to the value corresponding to the running value of $d_h x$.

The examples considered demonstrate that nonsynchronous movement of the fields is the response of the system to the drastic change in the process conditions, and the directionality of the response promotes restoration of the disturbed correlation between the concentration profiles in the gas and solid phases. Conversely, with constant external conditions, synchronous movement of the field ($\partial v/\partial h = 0$) is the most probable, and it is usually realized in practice.

For a period of a constant rate of drying, the latter can be proved analytically. Here, the air moisture content on the particle surface is constant and equal to x_{sat}. The local drying rate is expressed by the equation

$$-\frac{\partial u}{\partial \tau} = \beta f (x_{sat} - x)\,. \tag{14}$$

In view of Eq. (1), after integration we have

$$x = x_{sat} - (x_{sat} - x_0)\, e^{-ah}\,, \tag{15}$$

$$u = u_0 - \beta f (x_{sat} - x_0)\, e^{-ah}\,, \quad a = \frac{\beta f \rho_s (1 - \varepsilon)}{w \rho_g}\,. \tag{16}$$

Differentiating Eqs. (15) and (16) with respect to h, we take their ratio:

$$\left(\frac{\partial x}{\partial u}\right)_\tau = \frac{\partial x}{\partial h} \bigg/ \frac{\partial u}{\partial h} = \frac{1}{\beta f \tau}\,. \tag{17}$$

Hence, at constant τ, the value of the derivative is the same at all points of the bed, and, with consideration for Eq. (4), we have synchronous movement of the fronts. Determining the time from Eq. (16) and simultaneously solving Eqs. (15), (17), and (4), we again obtain Eq. (11).

The property of synchronism for a period of a falling rate is revealed in experiments. Here no substantial departures from the regularities established above were noted. To confirm the foregoing, Fig. 1 presents data of [4]. Vertical line segments characterizing the distance Δh that the fronts cover over the time $\Delta\tau$ are drawn between adjacent curves. Equality of the line segments attests to equality of the ascent velocities of the fronts. Thus, synchronism of the moving concentration fields to which Eq. (11) corresponds is a specific property of the filter beds in the case where the gas parameters are constant at the entrance.

Under the employed assumption of the absence of reverse mass transfer, the state of the upper fronts does not affect that of the lower ones, and therefore, the velocity of the latter does not depend on the layer thickness, and Eq. (11) is valid for beds of any thickness, no matter whether or not the gas really attains a saturation state at the exit.

It is of interest to analyze the process graphically in the coordinates $x-u$ (Fig. 2). In conformity with Eqs. (4) and (11), the operating lines of the process at various instants of time are straight lines turning clockwise around the pole 0 with the coordinates (u_0, x_{sat}). If there is a zone of a falling rate drying at the bottom, x_{sat} decreases as a result of additional heat consumption on material heating and removal of bound moisture, and therefore drift of the pole toward smaller x_{sat} (positions O-O_1-O_2) is observed. In such an event, the latter can be found using a transcendental equation characterizing enthalpy balance:

$$c_g \frac{t_0 - t(x_{sat})}{x_{sat} - x_0} = r + q_b(\dot{u}) + c_s \frac{\dot{\theta} - \theta_0}{u_0 - \dot{u}}. \tag{18}$$

The limiting position of the lines is A-O_2, at which $\dot{u} \to u_e$, and here the ascent velocity of the fronts is minimum and subsequently constant.

The property of synchronism allows a fairly simple solution of many problems on unsteady processes in a filter bed without resorting to full-scale modeling. The elementary distance travelled by the moving front is

$$dh = \bar{v}\, d\tau. \tag{19}$$

Because the front velocities are equal, the time interval $d\tau$ at any point of the bed can be expressed in terms of the rate of the process (drying) $\dot{u}$ on the LB at a given instant of time:

$$d\tau = d\dot{u}/\dot{u}'. \tag{20}$$

By simultaneously solving Eqs. (17), (20), and (11) and integrating we obtain an equation that characterizes the concentration distribution over the bed height:

$$h = \frac{w\rho_g}{\rho_s(1-\varepsilon)} \int_{u}^{\dot{u}} \frac{x_{sat} - x_0}{u_0 - \xi} \frac{d\xi}{\xi'(\xi)}. \tag{21}$$

Here, $\xi'(\xi) = \dot{u}'(\dot{u}) > 0$ is the familar dependence of the rate of the process on the moisture content at the LB of the bed.

Thus, the calculation by this method is performed in two steps. In the first step, the kinetic problem is solved for the LB: $\dot{u} = \varphi(\tau)$. Any known methods can be used here. Generally, this step presents no special difficulties since the process is examined at constant t_0 and x_0.

In the second step, the functions $\dot{u} = \varphi(\tau)$ and $\dot{u}'(\dot{u})$ that are found are utilized in integral (21). The solution obtained is exact; however, the presence of singular points at $\xi = u_0$ and $\xi = u_e$ should be taken into account. Direct use of the relations for the LB in Eq. (21) does not at all mean identity of the process kinetics on it and at the height h.

Clearly, the method permits the solutions for the concentration and temperature problems to be separated because only the concentration profile is found from Eq. (21). This is sufficient for most practical problems, and if temperature profiles are also required, appropriate heat balance equations are added. The problems of the first step are solved, as a rule, with allowance for the change in the temperature of the material.

The property of synchronism also implies an analogy between filter and countercurrently moving beds. Indeed, if the concentration fields move with the same velocity at all points, then, by imparting to the bed a velocity equal in magnitude and opposite in direction it is easy to fix the process in time and space with a constant moisture content at the exit. Therefore, Eq. (21) can also be applied to designing countercurrent devices.

All the reasoning above also holds for gas adsorption processes. Here, the expression for the front velocity of the fronts is modified somewhat:

$$\bar{v} = \frac{w\rho_g}{\rho_s(1-\varepsilon)} \frac{x_0 - x^*}{\dot{u} - u^*}. \tag{22}$$

Figure 3 presents operating lines of adsorption for two types of equilibrium sorption isotherms. For isotherms that are convex downward (Fig. 3a), the operating lines turn around a pole with the coordinates

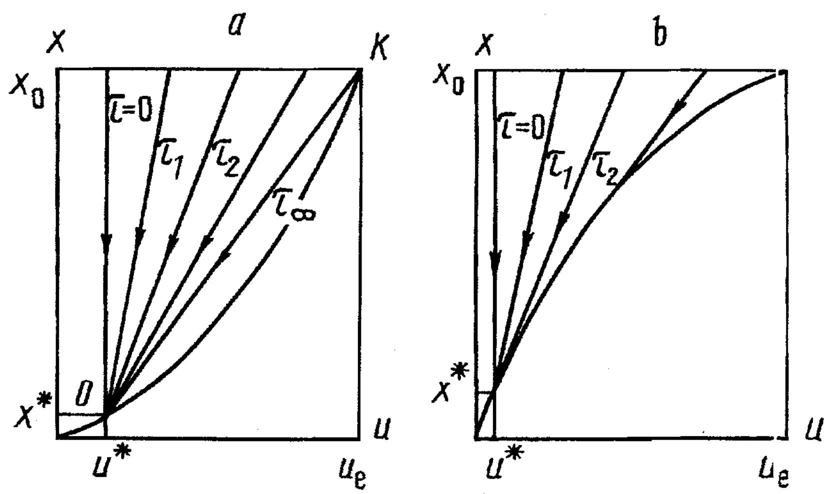


Fig. 3. Operating lines of adsorption for two types of equilibrium isotherms.

(x^*, u^*), and in this case the velocity of the fronts gradually decreases. The limiting position of the operating lines is OK, and it corresponds to a steady process. The moisture content distribution over the height at any instant of time can be determined from an equation of type (21), for which purpose information on just the sorption kinetics of sorbent particles on the LB is sufficient.

For an isotherm that is convex upward (Fig. 3b), a steady state is unattainable and the operating lines envelope the isotherm tangentially. Here, at points of tangency and to the left of them an equilibrium adsorption [9] is observed that is characterized by an infinitesimal driving force of external mass transfer and by disturbance of movement synchronism.

Unlike existing methods, the considered method of a synchronously moving field does not make it necessary to take into account the specific features of various zones over the bed height or to calculate the moving boundaries between them or the temperature fields, which facilitates calculations appreciably and improves their accuracy.

NOTATION

c, specific heat; f, specific surface of the product, m^2/kg; h, coordinate (height); r, heat of evaporation of the free liquid; q_b, energy of binding of the moisture with the material; t, gas temperature; u, moisture content of the product, kg of moisture/kg of dry product; v, velocity of movement of the fields; x, moisture content of the gas, kg/kg of dry gas; w, gas velocity per full cross section; β, interphase mass transfer coefficient; ε, bed porosity; Θ, product temperature; ρ, density; τ, time. Subscripts: g, gas; s, solid phase; sat, saturated; cr, critical; e, equilibrium; 0, parameters at the entrance; ′, parameters of the lower boundary; *, parameters of fresh sorbent.

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